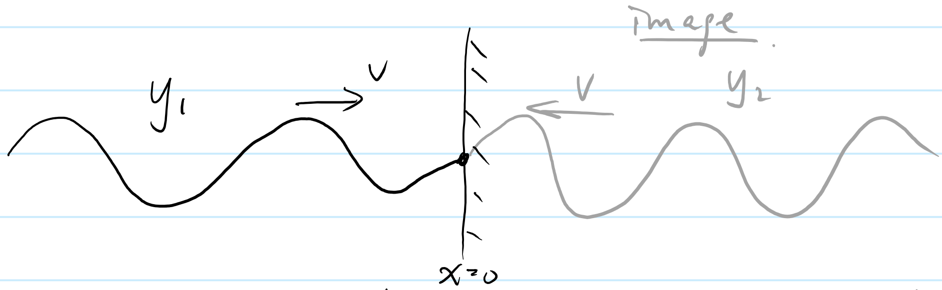
Wave and Sound II

Standing wave (from reflection)

- a superposition of two waves travelling in opposite directions.

Consider: a wave travelling to $+x$ and reflected by a wall at $x=0$.



Let the incident wave, $y_1 = A\cos(kx-\omega t)$

the reflected wave, $y_2 = -A\cos(kx+\omega t)$

up side down. left moving

The resulting wave is $y = y_1 + y_2$

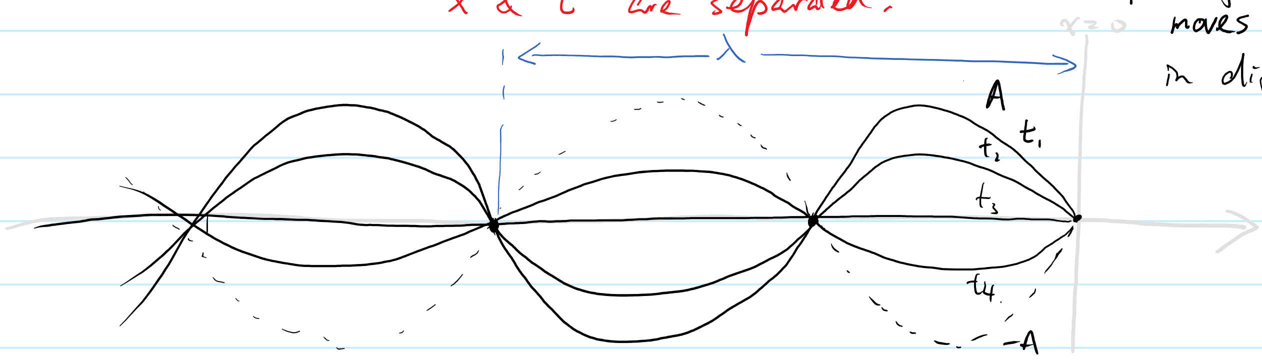
$$y = A[\cos(kx-\omega t) - \cos(kx+\omega t)]$$

$$= -2A \sin\left(\frac{kx-\omega t+kx+\omega t}{2}\right)\sin\left(\frac{kx-\omega t-kx-\omega t}{2}\right)$$

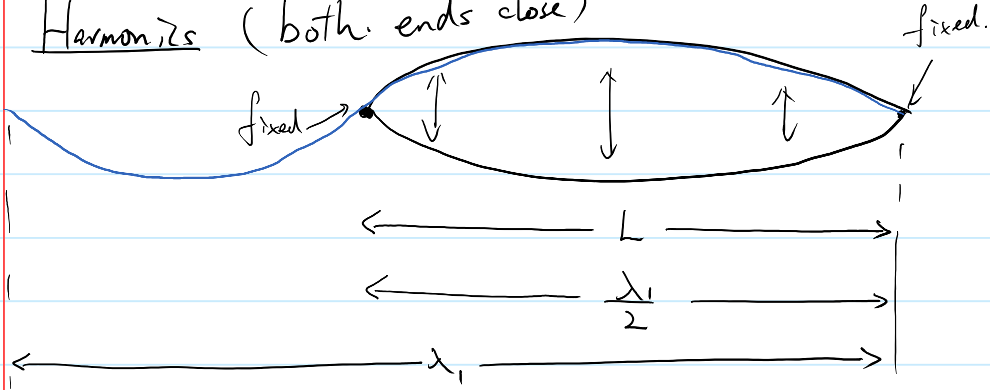
$$= -2A\sin(kx)\sin(-\omega t)$$

$$= 2A\sin(kx)\sin(\omega t) = \underbrace{\overbrace{A(x)}^{\text{Amplitude}} \cdot \overbrace{\sin(\omega t)}^{\text{SHM}}}_{\text{each part of the wave moves in SHM in different amplitude}}$$

x & t are separated.



Harmonics (both ends close)

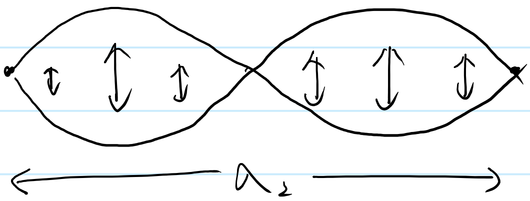


First Harmonic
(longest wavelength)

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

freq. depends on L.

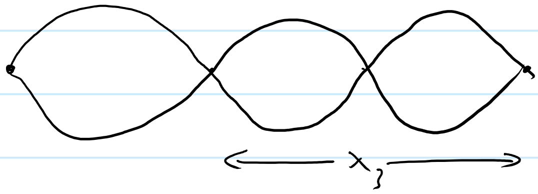
$$L = 2\left(\frac{\lambda_2}{2}\right)$$



Second Harmonic

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2\,\frac{v}{2L}$$

$$L = 3\left(\frac{\lambda_3}{2}\right)$$



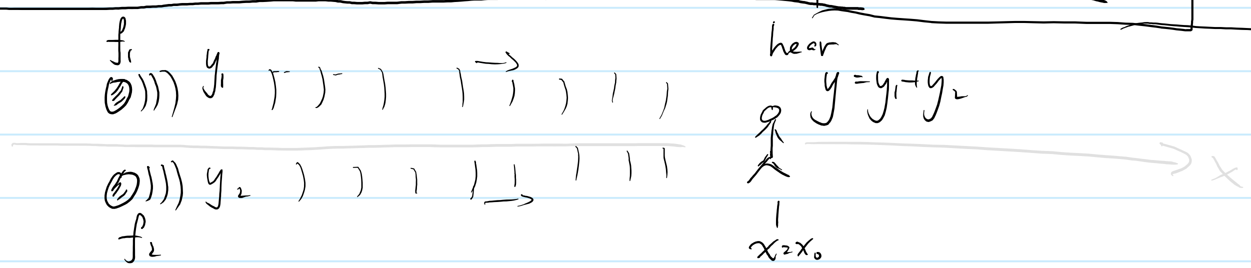
Third Harmonic

$$f_3 = \frac{v}{\lambda_3} = 3\,\frac{v}{2L}$$

n-th Harmonic

$$\boxed{f_n = n \cdot \frac{v}{2L}}$$

Beats.



$f_1 \approx f_2$ but different.

Let $\Delta f = f_1 - f_2$, $\Delta f \ll f_1 \approx f_2$ e.g. $f_1 = 401\,Hz$, $f_2 = 400\,Hz$.
$\Delta f = 1\,Hz \ll 400\,Hz$.

at the position of the listener. $x = x_0$
the superposition of the wave is

$$y = y_1 + y_2 = A\cos(\underbrace{k_1x_0}_{\text{constant}} - \omega_1 t) + A\cos(\underbrace{k_2x_0}_{\text{constant}} - \omega_2 t)$$

Let $k_1x_0 = \phi_1$ $\qquad$ $k_2x_0 = \phi_2$

$$y(x_0,t) = A\cos(-\omega_1 t+\phi_1) + A\cos(-\omega_2 t+\phi_2) \qquad \omega_1 = 2\pi f_1,\ \omega_2 = 2\pi f_2$$

$$= 2A\cos\left(\frac{-\omega_1 t+\phi_1+(-\omega_2)t+\phi_2}{2}\right)\cos\left(\frac{-\omega_1 t+\phi_1-(-\omega_2 t)-\phi_2}{2}\right)$$

$$= 2A\cos\left(\frac{-(\omega_1+\omega_2)t+\phi_3}{2}\right)\cos\left(\frac{-(\omega_1-\omega_2)t+\phi_4}{2}\right), \quad \phi_3 = \phi_1+\phi_2,\ \phi_4 = \phi_1-\phi_2$$

$$= 2A\cos\left(\frac{-2\pi(f_1+f_2)t}{2}+\phi_3'\right)\cos\left(\frac{-2\pi(f_1-f_2)t}{2}+\phi_4'\right)$$

$$= 2A\cos\underbrace{\left(-2\pi f_{avg}t+\phi_3'\right)}_{\text{periodic function with frequency } = f_{avg} = \frac{f_1+f_2}{2}}\ \cos\underbrace{\left(-2\pi\left(\frac{\Delta f}{2}\right)t+\phi_4'\right)}_{\text{periodic function with frequency } \left(\frac{\Delta f}{2}\right)}$$

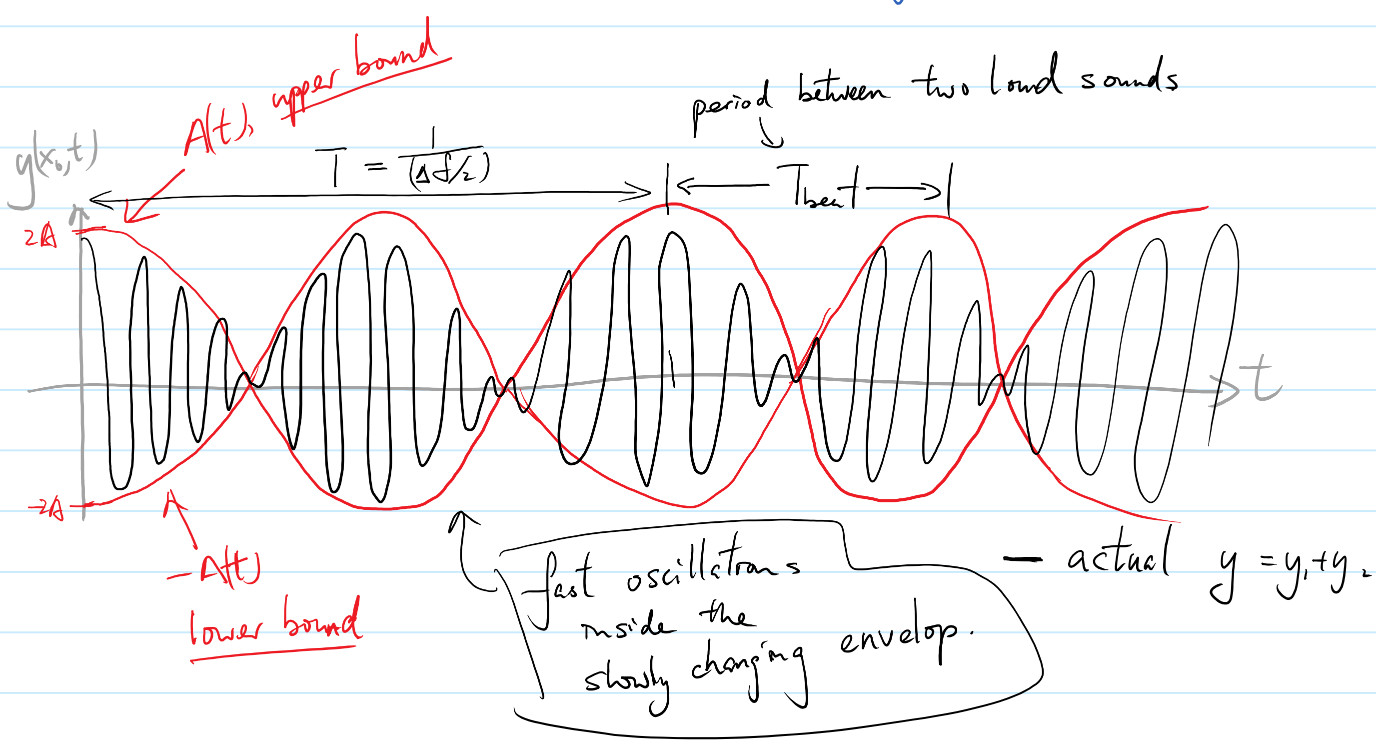
f_{avg}: fast

$\left(\frac{\Delta f}{2}\right)$: slow.

$$= \underbrace{\underbrace{2A\cos\left(-2\pi\left(\frac{\Delta f}{2}\right)t\right)}_{\text{slowly changing in time}}}_{\text{Consider as a time dependent amplitude}} \cdot \underbrace{\underbrace{\cos\left(-2\pi f_{avg}t\right)}_{\text{quickly changing in time.}}}_{\text{a fast oscillation under the amplitude } A(t)}$$

dropped ϕ_4', ϕ_3' for simplicity.

$$= A(t)\cdot\cos\left(-2\pi f_{avg}t\right)$$



$$T_{beat} = \frac{1}{2}T = \frac{1}{2}\frac{1}{\left(\frac{\Delta f}{2}\right)} = \frac{1}{\Delta f}.$$

Beat frequency (frequency of the loud sounds)

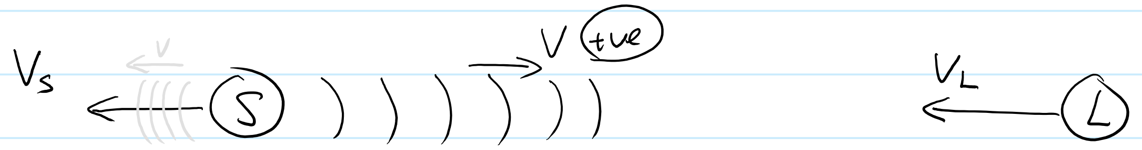
$$f_{beat} = \frac{1}{T_{beat}} = \Delta f$$

Doppler Effect of sound

Frequency of sound changes when the source and/or the observer (listener) are moving.

A referential situation.

- both the source (S) and the listener (L) are moving in opposite direction of the sound wave propagation.



Consider 3 reference frames: source, ground, listener.

	(L)	Ground	(S)
Wave speed:	$\vec{V} - \vec{V}_L \rightarrow V - (-V_L) = V + V_L$	V	$\vec{V} - \vec{V}_s \rightarrow V + V_s$
freq.	f_L	f_G	f_S
$\lambda = \frac{V}{f}$	$\lambda_L = \frac{V+V_L}{f_L}$	$\lambda_G = \frac{V}{f_G}$	$\lambda_S = \frac{V+V_s}{f_S}.$

Key step: Wavelength of the wave must be the same measured by all initial observers!

$\because$ 1 meter is 1 meter no matter who measures it.

$$\Rightarrow \quad \lambda_L = \lambda_G = \lambda_S$$

$$\frac{V+V_L}{f_L} = \frac{V+V_S}{f_S}$$

$$\Rightarrow \quad \boxed{f_L = \frac{V+V_L}{V+V_S} f_S.}$$

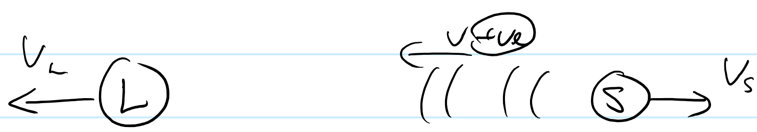
for the case:

$$\vec{V}_S \to +V_S$$

$$f_L = \frac{V+V_L}{V-V_S} f_S$$

↑ change sign. $\because$ speed of wave measured by S is slower.

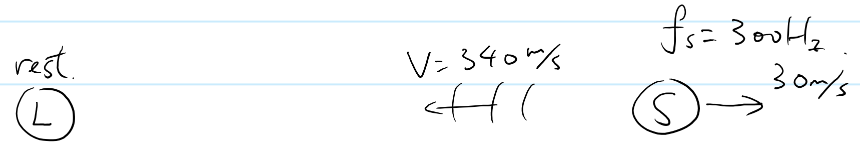
or.



$$f_L = \frac{V-V_L}{V+V_S} f_S.$$

← speed of wave measured by L is slower.

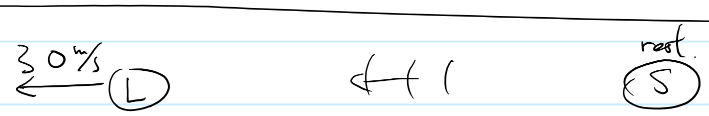
Example ①



$$f_L = \frac{V + V_L}{V + V_s} f_s \qquad V_L = 0, \quad V_s = 30$$

$$f_L = \frac{340}{370} 300 = 276 \text{ Hz}$$

②

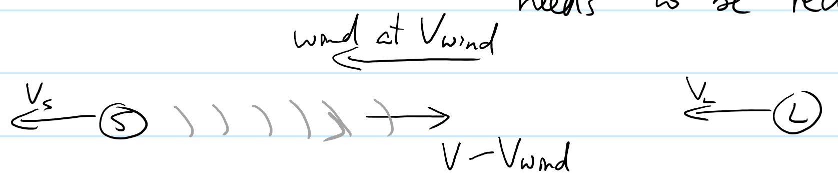


$$f_L = \frac{V - |V_L|}{V} f_s = \frac{310}{340} 300 = 274 \text{ Hz.}$$

Same relative velocity between S and L, but different results.

$\Rightarrow$ velocity of both L & S relative to the ground are important.

What if there is wind? $\Rightarrow$ speed of wave relative to L & S needs to be reconsidered,



Sound wave travels in the air.
Wind = air movement.

$\Rightarrow$ Speed of sound wave relative to the ground changes. as $V - V_{wind}$

Wave Speed relative L = $V - V_{wind} + V_L$
 " " " S = $V - V_{wind} + V_s$

$$\Rightarrow f_L = \frac{V - V_{wind} + V_L}{V - V_{wind} + V_s} f_s.$$